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Accurate Positional and Thermal Parameters of Hexamethylenetetramine from *K*-Shell X-ray Diffraction Data

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Geometric and thermal parameters for hexamethylenetetramine have been determined from 95 unique reflections (Mo $K\alpha$, graphite monochromator) in the $\sin \theta/\lambda$ range 0.65–1.10 \AA^{-1} . Both standard (second cumulant) anisotropic thermal parameters and third cumulant parameters as well as positional parameters for C and N are in excellent agreement with the corresponding neutron parameters. H atom parameters were not determined from the X-ray data. The estimated standard deviations for the X-ray parameters are generally one fifth (or less) of those for the neutron parameters. This study shows that with X-ray data it is possible to determine geometric and thermal parameters free of effects from valence electron asphericity.

Introduction

Recent interest in the experimental determination of molecular electron density distributions has renewed

interest in the comparison of X-ray and neutron diffraction results. The aspherical electron distribution of atoms in molecules will influence parameters refined from conventional X-ray data. In a survey, Hamilton (1969) found significant differences between X-ray and neutron results in all cases.

When the positional and thermal parameters deter-

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mined from neutron diffraction are used with X-ray data to calculate a difference electron density map, the difference density represents the redistribution which occurs as a result of bonding.

An alternative approach in determining electron density distributions is to use parameters refined from high-order X-ray data rather than neutron parameters (Stewart, 1968). An inspection of atomic scattering factors suggests that C and N valence electrons will not contribute significantly to X-ray scattering above 0.65 \AA^{-1} ($\sin \theta/\lambda$). If the core electrons are not significantly affected by bonding and if bonding features do not contribute to high-order scattering, the results of an accurate high-order refinement should be free of bias from the aspherical distribution of valence electrons and should show no systematic differences from neutron diffraction results.

X-ray data have been collected from a single crystal of hexamethylenetetramine (HMT) to test the hypothesis that high-order refinements of X-ray data will yield the same results as neutron diffraction data. The accurate neutron diffraction study by Duckworth, Willis & Pawley (1970) is available for comparison.

Experimental

Crystals of hexamethylenetetramine (J. T. Baker Chemical Co.) were grown from a 95% ethanol solution in a convection tube (Hope, 1971*b*). A crystal was ground into a sphere of diameter 0.45 mm, coated with a thin layer of glue to prevent sublimation, and mounted on a Picker automated four-circle diffractometer equipped with an incident-beam graphite monochromator. The angular coordinates ($\chi, \omega, 2\theta$) of 15 Mo $K\alpha_1$ reflections were used in a least-squares calculation to determine the crystal orientation and cell dimensions. The cell edge was found to be $a_0 = 7.028(2) \text{ \AA}$ at 24°C with $\lambda = 0.70926 \text{ \AA}$ for Mo $K\alpha_1$ radiation.

The positional and thermal parameters determined by neutron diffraction (Duckworth *et al.*, 1970) were used together with sample scan and background measurements to predict which reflections would have measurable X-ray intensities. The setting angles χ , φ , and ω were chosen to minimize the contributions of multiple Bragg scattering using a procedure similar to that of Coppens (1968). For each reflection, a rotation ψ about the diffraction vector was chosen which would avoid contact of all other observed reflections with the sphere of reflection.

The intensities of 1251 selected reflections were measured using a $\theta:2\theta$ scan, graphite monochromatized Mo $K\alpha_1$ radiation, and a scintillation detector with a pulse-height analyzer. Each reflection was scanned from $[2\theta(\alpha_1) - 0.8^\circ]$ to $[2\theta(\alpha_2) + 0.7^\circ]$ with a scan speed of $0.25^\circ(2\theta)/\text{min}$. Background counts were taken for 100 s at each end of the scan range. Coincidence losses were minimized by adding an attenuator filter when the count rate exceeded 10000 c.p.s.

Two check reflections were measured after every 50 reflections and showed a gradual decline in intensity, probably caused by sublimation, amounting to 10% at the end of data collection. A correction for this was obtained from a single linear least-squares fit of the normalized intensities of both check reflections. Each recorded number of counts, N , was assigned a standard deviation $\sigma(N) = [N + (0.0065N)^2]^{1/2}$; the factor 0.0065 reproduces the observed variance in the check reflections after correction for decay. Based on the values of $\sigma(N)$ for scan and background counts, e.s.d.'s for the net intensities were calculated.

Lorentz and polarization corrections were applied to the net intensities. The polarization factor includes the polarization by the monochromator crystal (Hope, 1971*a*). Each measured intensity I_{obs} was corrected for first (α_1) and second order (α_2) thermal diffuse scattering included in the scan

$$I_{\text{Bragg}} = I_{\text{obs}} / (1 + \alpha_1 + \alpha_2).$$

The correction was calculated using a procedure described by Stevens (1974) which includes the $K\alpha_1$, $K\alpha_2$ wavelength distribution, the elastic anisotropy, and the orientation of the crystal defined by the χ , φ , and ω angles. The elastic constants of HMT have been measured by Haussühl (1958). The horizontal and vertical detector slits each subtended an angle of 1° . Each correction was calculated with a precision of 0.1%. The largest value was 0.30 for α_1 and 0.07 for α_2 .

At least six symmetry equivalent forms of each independent reflection were measured. The intensities and e.s.d.'s of the symmetry equivalent forms were used to calculate a weighted average intensity, e.s.d. of the mean, as well as external ('observed') standard deviation for each independent reflection. For most reflections, the external e.s.d.'s were less than the average e.s.d.'s from counting statistics. However, for weak and very strong reflections the external e.s.d.'s were larger.

Refinement

The quantity $\sum w(F_{\text{obs}} - K|F_{\text{cal}}|)^2$ was minimized in a full-matrix least-squares refinement, with $w = 1/\sigma^2$ where σ is the e.s.d. for the mean of each observed structure factor. The data were divided into two groups: a low-order group with $0.0 < \sin \theta/\lambda \leq 0.65 \text{ \AA}^{-1}$, and a high-order group with $0.65 < \sin \theta/\lambda < 1.10 \text{ \AA}^{-1}$. Both groups were refined separately.

Refinements were performed with two descriptions of the thermal motion, (1) with conventional anisotropic thermal parameters where the structure factor is given by

$$F_c^{(2)} = \sum_p f^{(p)} \exp(2\pi i x_i^{(p)} h_i) \exp[-\frac{1}{4}(B_{ij} a_i^* a_j^* h_i h_j)],$$

and (2) with third cumulant thermal parameters (Johnson, 1969), where the structure factor is given by

$$F_c^{(3)} = \sum_p F_c^{(2,p)} \exp(-i C_{ijk}^{(p)} a_i^* a_j^* a_k^* h_i h_j h_k) \quad (i, j, k = 1, 2, 3),$$

where \sum_p implies summation over all atoms in the unit cell, the subscripts i , ij and ijk imply single, double and triple summation, respectively and $F_c^{(2,p)}$ is the single-atom contribution to $F_c^{(2)}$. The other symbols have their conventional meanings.

Refinements of type (1) were carried out with both relativistic Hartree-Fock (RHF) and Slater-type orbital form factors.

Scattering factors calculated by Doyle & Turner (1968) were used for nitrogen and carbon. The contracted spherical scattering factor calculated by Stewart, Davidson & Simpson (1965) was used for hydrogen.

During initial refinement, inspection of the deviations between F_o and F_c indicated secondary extinction in the 011 reflection, which has an intensity more than four times greater than the intensity of the next strongest reflection. The reflection was not included in further refinements. All other reflections were considered to be 'observed'.

Since hydrogen does not scatter sufficiently at high angles hydrogen parameters in the high-order refinement were fixed at the values determined by neutron diffraction. The results of type (1) refinements of the X-ray data are compared with the neutron diffraction results in Table 1. Molecular optimized scattering factors (MOSTO) for carbon and nitrogen were calculated from Slater-type orbital exponents optimized to the methane and ammonia molecules, respectively, by Hehre, Stewart & Pople (1970).

The results of refinement of the X-ray data with third-cumulant thermal parameters are compared with a third-cumulant refinement of the neutron data in Table 2. A significant improvement in the model results from third cumulant refinements for both high- and low-order data, as judged by Hamilton's (1965) R -factor ratio test.

The parameters from each of the X-ray refinements are compared with the neutron diffraction results using the chi-squared test of Hamilton (1969). Included in each table is the value of χ^2 required for differences in the parameters to be significant at the 0.05 level.

Table 3 gives a comparison of F_o and F_c . The calculated structure factors are based on the high-order refinement with standard anisotropic temperature factors, using RHF form factors.

Discussion

The differences between the low-order X-ray parameters and the neutron parameters are significant and can be attributed to the aspherical distribution of valence electrons. Surprisingly, in our study the nitrogen is not displaced towards the lone pair, in contrast to the results of the previously reported X-ray work (Becka & Cruickshank, 1963).

The hypothesis that the differences between the high-order X-ray parameters and neutron parameters are due to random errors cannot be rejected as indicated by the χ^2 test. In this structure, the nuclear positions and thermal parameters coincide with those

Table 1. Comparison of refinements with standard (second-cumulant) temperature factors

Form factor	RHF	RHF	MOSTO	MOSTO	Neutron
Sin θ/λ range (\AA^{-1})	0.0-0.65	0.65-1.10	0.0-0.65	0.65-1.10	0.0-0.8
Number of parameters	14	8	14	8	14
Number of reflections	54	95	54	95	49
R (%)	2.86	6.35	2.49	6.31	2.33
R_w (%)	3.55	3.76	3.23	3.68	
Goodness of fit	12.96	3.31	11.74	3.41	
Scale for F_o	0.0258 (1)	0.0280 (1)	0.0280 (1)	0.0279 (1)	
χ_{obs}^2	187.1	4.4	185.5	5.2	
$\chi_{0.05}^2$	22.4	14.1	22.4	14.1	
$(\varphi_X - \varphi_N)_{r.m.s.}$ *	1.62°	1.88°			
Nitrogen					
x, y, z	0.1223 (1)	0.1223 (1)	0.1221 (1)	0.1223 (1)	0.1222 (2)
$B_{11, 22, 33}$	4.13 (3)	3.84 (2)	3.74 (2)	3.80 (2)	3.87 (6)
$B_{12, 13, 23}$	-0.79 (2)	-0.83 (1)	-0.77 (1)	-0.83 (1)	-0.81 (6)
Carbon					
x	0.2382 (1)	0.2378 (1)	0.2384 (1)	0.2378 (1)	0.2370 (5)
B_{11}	2.53 (2)	2.11 (2)	2.34 (2)	2.08 (2)	2.04 (12)
$B_{22, 33}$	5.40 (3)	5.04 (3)	5.25 (3)	5.01 (3)	5.11 (12)
B_{23}	0.27 (4)	0.04 (3)	-0.35 (4)	0.04 (3)	0.02 (20)
Hydrogen					
x, y	0.0898 (8)	[0.0909]	0.0905 (7)	[0.0909]	0.0909 (6)
z	-0.3187 (7)	[-0.3266]	-0.3215 (7)	[-0.3266]	-0.3266 (7)
$B_{11, 22}$	7.36 (36)	[8.77]	6.73 (32)	[8.77]	8.77 (28)
B_{33}	3.42 (21)	[4.07]	3.68 (20)	[4.07]	4.07 (22)
B_{12}	-0.66 (32)	[-0.42]	-1.43 (27)	[-0.42]	-0.42 (32)
$B_{13, 23}$	4.06 (21)	[1.88]	4.43 (19)	[1.88]	1.88 (16)

* φ_X and φ_N are X-ray and neutron phase angles respectively.

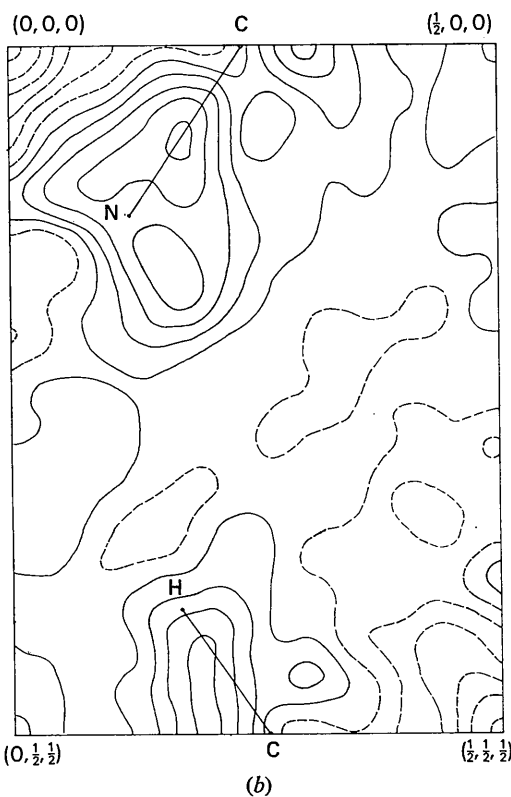
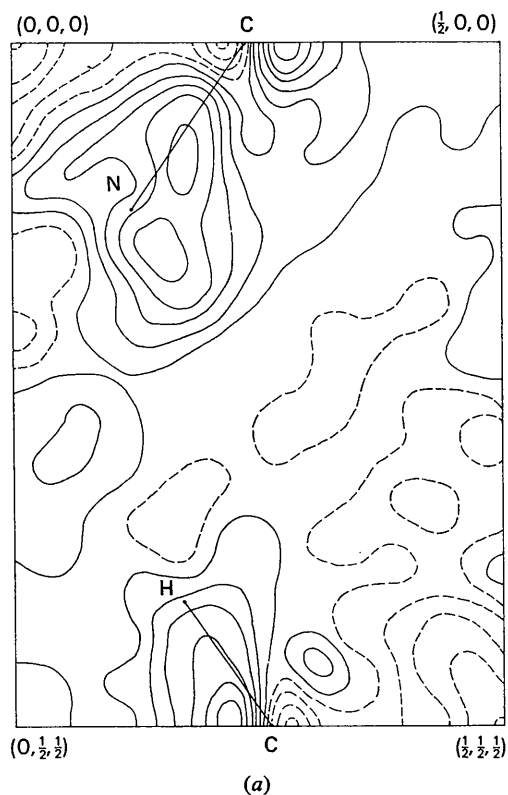


Fig. 1. Difference electron density, 110 plane (a) X-N difference map. (b) X-X (high-order) difference map.

axis lies in the plane and passes through the upper left corner and lower right corner.

The lone-pair density of the nitrogen atom is clearly visible extending from the nitrogen along the threefold axis. Bonding density is also present in the carbon-nitrogen and carbon-hydrogen bond region.

A difference density map calculated with the X-ray data and the positional and thermal parameters from the high-order X-ray refinement is plotted in Fig. 1(b). Since there is no significant difference in the results of the neutron and the high-order X-ray refinements, the difference density maps are nearly identical.

HMT has been used in a joint refinement of X-ray and neutron data in which separate positional parameters and common thermal parameters are refined (Duckworth, Willis & Pawley, 1969). Since the thermal parameters obtained from low-order X-ray data show greater bias due to the valence electrons than the positional parameters, these results indicate that a joint refinement with common positional parameters and separate thermal parameters would be more appropriate, although, in our opinion, the exact interpretation of the results of such refinements would be so difficult as to render them nearly useless. We do not recommend this procedure.

In this case it has been possible, using *K*-shell X-ray diffraction data, to obtain accurate positional and thermal parameters which are not biased by the aspherical distribution of valence electrons. In other cases (e.g. structures with oxygen atoms) scattering from lone-pair electrons may extend beyond a $\sin \theta/\lambda$ value of 0.65 \AA^{-1} , requiring higher-order data.

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